PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION FOR RECRUITMENT TO THE POSTS OF PROVINCIAL MANAGEMENT SERVICE, ETC. - 2014

MATHEMATICS (OPTIONAL) PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

Note: Attempt any THREE questions form Section "A" and TWO questions from Section "B". Simple Calculator is allowed.

SECTION A

Q.1. (a) Show that $\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x}=e$ as x approaches infinity while taking on positive or negative real values.

(b) Prove that if f(x) is continuous on the interval $a \le x \le b$, and if c is any number between a and b and $f(c) > 0$, then there exists a number $\lambda > 0$ such that whenever $c-\lambda < x < c+\lambda$, then $f(x) > 0$.

Q.2. (a) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.

(b) Find the length of the curve

$$x=\int_0^y \sec^4 t-1dt, -\pi/4 \le y \le \pi/4.$$

Q.3. (a) Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the (i) height and (ii) radius changing when the pile is 4 m high? Answer in cm/min.

(b) Find the center of mass of a thin plate of constant density δ covering the region bounded above by the parabola $y=4-x^2$ and below by the x-axis.

Q.4. (a) Find an equation of the straight line joining two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angles are given. Hence find equations of the tangent and normal at any point θ on the ellipse.

(b) Evaluate $\int_0^3 (2x-x^2)dx$, taking 10 intervals by using

(i) Simpson's 1/3rd rule

(ii) Simpson's 3/8th rule

Q.5. (a) Solve the initial value problem:

$$(4x^3e^{x+y}+x^4e^{x+y}+2x)dx+(x^4e^{x+y}+2y)dy=0, y(0)=1$$

(b) The line normal to a given curve at each point (x, y) on the curve passes through the point (2, 0). If the curve contains the point (2, 3), find its equation.

SECTION "B"

Q.6. (a) Let $a_n > 0, n=1,2,3,......$ Prove that the alternating series

$$a_1-a_2+a_3-...+(-1)^{n-1}a_n+.....$$

converges if the following two conditions hold:

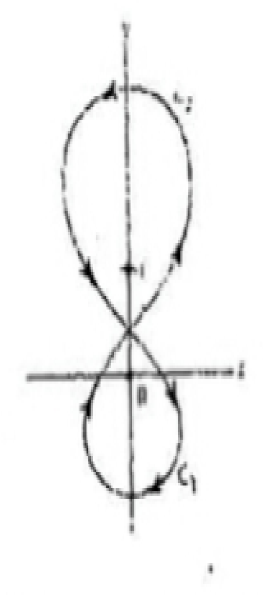
(i) $\{a_n\}$ is a non increasing sequence, $i,e., a_1 \ge a_2 \ge a_3 \ge .....$

(ii) $\lim_{n\to\infty} a_n = 0.$

(b) Determine the values of x for which the power series $\sum_n^{\infty}=2\frac{x^n}{\ln n}$ converge absolutely, converges conditionally and diverges.

Q.7. (a) Evaluate $\oint_c \frac{dz}{z^2+1}$, where C is the ci $|z|=4$.

(b) Evaluate $\oint_c \frac{z^3+3}{z(z-i)^2}dz$ where C is the figure - eight contour shown in figure.



Q.8. (a) Find an equation to the cone whose vertex is the origin and directrix is the circle $x=a, y^2+z^2=b^2$ Show that the trace of the cone in a plane parallel to the xy plane is a hyperbola.

(b) Show that the torsion of the binormal indicatrix $x_3=b(s)$ of sufficiently regular curve is $\tau_3=\frac{\tau k-k\tau}{\tau(k^2+\tau^2)}$.

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MATHEMATICS (OPTIONAL) PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

Note: Attempt 'Five' questions in all by selecting a' ... questions from each sections. All questions carry equal marks. Calculator is all...

SECTION A

Q.1. (a) Define a subgroup and prove that a nonempty subset H of a group G is a subgroup of G if and only if for each pair $a,b \in H, ab^{-1} \in H$.

(b) Distinguish between normalizer and centralizer of a subset X of a group G. Find the conditions on each of X and G for coinciding both of these concepts.

Q.2. (a) Distinguish between Normal and Characteristic subgroups. Prove or disprove that every normal subgroup is a characteristics subgroup.

(b) Define automorphism of a group. Let G be a group with Z (G) as its Centre and I (G) the group of Inner automorphisms. Then show that $G/Z(G) \cong I(G)$.

Q.3. (a) Distinguish between Integral domain and Field. Give an example of non-commutative ring.

(b) If U and W are finite dimensional subspaces of a vector space V over a field F, then prove that $\dim(U+W)=\dim(U)+\dim(W)-\dim(U\cap W)$.

Q.4. (a) Define field extension and illustrate with examples of finite and infinite fields.

(b) Show that the vectors (1-i, i) and (2, -1 + i) in C^2 are linearly dependent over C but linearly independent over R.

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SECTION - B

Q.5. (a) Distinguish between the following terms: Metric space and topological space, closed and open sphere, Interior and exterior of a set.

(b) Define convergence of a sequence and prove that a sequence in a metric space converges to exactly one point.

Q.6. Using Gram-Schmit process of orthonormalization transform the basis {(1, -1, 0), (2, -1, -2), (1, -1, -2)} into orthonormal basis.

Q.7. Find a real orthogonal matrix P for which $P^{-1}AP$ is diagonal where $A=\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$.

Q.8. (a) Prove that the eigenvalues of a symmetric matrix are all real.

(b) Solve the following system of equations by Gaussian elimination method:

$$x_1+5x_2+2x_3 = 9$$

$$x_1+x_2+7x_3 = 6$$

$$-3x_2+4x_3 = -2$$