PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION FOR RECRUITMENT TO THE POSTS OF PROVINCIAL MANAGEMENT SERVICE, ETC. - 2015

SUBJECT: MATHEMATICS (OPTIONAL) PAPER-I

TIME ALLOWED: THREE HOURS　　　MAXIMUM MARKS: 100

Note: Attempt any THREE questions from Section 'A' and TWO questions from Section 'B'. Each question carries equal marks. Simple Calculator is allowed.

SECTION A

1. (a) Distinguish between Rolle's theorem and Mean Value Theorem.

   If $f(x) = x(x-1)(x-2)$, $a = 0$ $b = \frac{1}{2}$ then find c of the Mean Value theorem.

   (b) Evaluate $\int_0^{\frac{\pi}{2}} \ln(\sin x)\,dx$

2. (a) With $n=$ find the area under the semi-circle $y = \sqrt{4-x^2}$ and above the x-axis by

   (b) Simpson's Rule (ii) The trapezoidal Rule, find the actual area as well.

   (c) Find the volume of tetrahedron enclosed by the plane $x + 2y + z = 2$ and the coordinate planes.

3. (a) Solve the following differential equations

   (i) $\frac{dy}{dx} + \frac{xy}{1-x^2} = xy^{\frac{1}{2}}$ (ii) $y'' - 3y' + 2y = 2x^3 - 9x^2 + 6x$

   (b) Find the area of the portion of the cone $x^2 + z^2 = 3z^2$ lying above the xy plane and inside the cylinder $x^2 + y^2 = 4y$.

4. (a) The population of a certain country is known to increase at a rate proportional to the number of people presently living in the country. If after two years the population has doubled, and after three years the population is 20,000, estimate the number of people initially living in the country.

   (b) Find an equation of tangent lines to the hyperbola $x^2 - y^2 = 16$ that pass through the point (2, -2).

5. (a) A mass of 2kg is suspended from a string with a known spring constant of 10N/m and allowed to come to rest. It is then set in motion by giving it an initial velocity of 150cm/sec. Find an expression for the motion of the mass assuming no air resistance.

   (b) Find the volume in the first octant between the planes $z = 0$ and $z = x + y + 2$, and inside the cylinder $x^2 + y^2 = 16$.

SECTION "B"

6. (a) Define convergence of series. Also state and prove Cauchy's integral Test for convergence of series.

   (b) Determine whether the series $\sum_{n=0}^{\infty} \frac{(-1)^{n-1}(n+2)}{n(n+3)}$ absolutely converges, conditionally converges of diverges.

7. (a) Find the interval and radius of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+1)^{2n}}{(n+1)^2 5^n}$$

   (b) Determine the centre, foci and vertices of the hyperbola $y^2 - x^2 - 10y + 6x = 0$ and sketch the graph.

8. (a) Distinguish between

   (i) Unit Normal and Unit binormal

   (ii) Curvature and torsion

   (iii) Osculating and rectifying planes.

   (iv) Prove that $\frac{d\hat{n}}{ds} = \tau b - k\hat{t}$

   (b) Evaluate $\oint_c \frac{dz}{1+z^2}$, where C is that part of the parabola $y = 4 - x^2$ from A (2,0) to B(-2,0).

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SUBJECT: MATHEMATICS (OPTIONAL) PAPER-II

TIME ALLOWED: THREE HOURS　　　MAXIMUM MARKS: 100

Note: Attempt any FIVE question by selecting at least TWO questions from each section. All question carry equal marks. Calculator is allowed.

SECTION-A

1. (a) Define order of an element of a group. Also prove that the order of an element of a finite group divides the order of the group.

   (b) Let H, K be sub-groups of a group G. Show that HK is a sub-group of G if and only if HK = KH.

2. (a) Let H, K be normal sub-groups of a group G and $H \subseteq K$. Show that (G/H) ϕ (K/H) ≅ G/K HG/KH

   (b) State and prove Lagrange theorem.

3. (a) Prove or disprove that a finite integral domain is a field.

   (b) Give an example of a finite ring which is not an integral domain.

4. (a) Does the set of all solution of the differential equation.

   $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

   Form a real vector space.

   (b) Let V be the vector space of all functions defined on R to R. Check whether the vectors 2, $4\sin^2 x$, $\cos^2 x$ are linearly independent in V.

SECTION-B

5. (a) Distinguish between the following terms:

   (i) Metric Space and Topological

   (ii) Closed and Open Sphere

   (iii) Interior and Exterior of a set

   (b) Show that the limit of a Convergent sequence in a metric space is unique.

6. (a) Show that {(1, 1, 1), (0, 1, 1), (0, 0, 1)} is a basis of R^3. Using Gram-Schmidt orthonormalization process, transform this basis into an orthonormal basis.

   (b) Show that the product and inverses of orthogonal matrices are orthogonal.

7. Solve the following system of equations by Gauss Siedal method:

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35$$

8. Find the eigenvalues and corresponding eigenvetors of the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$