PROVINCIAL MANAGEMENT SERVICE, ETC - 2016

SUBJECT: MATHEMATICS (PAPER-I)

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt FIVE Questions in All. THREE Questions from Section "A" and TWO Questions from Section "B". Calculator is allowed.

SECTION - A

Q No. 1: (a) Evaluate $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ where n tends to infinity through positive integral values only.

(b) State and prove Leibniz theorem. Find nth derivative of $e^x \ln x$

(20 Marks)

Q No. 2: a) Find (i) $\lim_{x \to 0} (\cot x)^{\sin 2x}$

(ii) $\lim_{x \to \frac{\pi}{2}} \left(\frac{\tan x}{\tan 3x}\right)$

b) Evaluate (i) $\int \frac{dx}{\sqrt{x} - x^{\frac{1}{3}}}$ (ii) $\int_{-3}^{3} |x|\, dx$

(20 Marks)

Q No. 3: a) Evaluate $\int_0^1 \int_0^x (x^2 + 4xy)\, dy\, dx$

b) Evaluate $\int\int\int_s 3(x^2 + y^2 + z^2)\, dv$

Where s is bounded by the planes $x = 1, x = 3, y = -1, y = 1$ $z = 2$ and $= 4$.

(20 Marks)

Q No. 4: **Solve the initial value problems:-**

a) $(x^2 + 3y^2)dx - 2xy\, dy = 0$, $y(2) = 6$

b) $(x^2 + 1)\frac{dy}{dx} + 4xy = x$, $y(2) = 1$

(20 Marks)

Q No. 5: a) A body of mass 2 slugs is dropped with no initial velocity and encounters an air resistance that is proportional to the square of its velocity. Find an expression for the velocity of the body at any time.

(b) Approximate $\int_0^2 \frac{1}{\sqrt{(x^2+1)}}\, dx$ with $n = 4$

By using rules (i) Trapezoidal (ii) Sampson's

(20 Marks)

P.T.O

SECTION - B

Q No. 6: a) Determine whether the series

$\sum_1^\infty \frac{arc\ tan\ x}{(1+x^2)}$ converges or diverges.

b) Distinguish between conditionally convergent and absolutely convergent series . Prove or disprove that an absolutely convergent series is convergent

(20 Marks)

Q No. 7: a) Find the interval and radius convergence of the series

$$\sum_{2}^{\infty} \frac{x^n}{(\ln \underset{n}{\cancel{x}})^n}$$

b) Show that $x^2 + 4y^2 - 4x + 8y + 4 = 0$ represents an ellipse. Sketch its graph and find its foci and vertices.

(20 Marks)

Q No. 8: (a) Evaluate $\oint_c \frac{3z^2+z}{z^2-1}\, dz$ where c is the circle $|z-1| = 1$.

(b) State and prove Serret-Frenet Formulae

(20 Marks)