MATHEMATICS (OPTIONAL) PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt any THREE questions from Section "A" and TWO questions from Section "B".

SECTION "A"

Q.1 (a) Draw the graph of the function $f(x)=\frac{|x|}{x}$ and discuss its continuity at $x=0$ 6

(b) Assume that oil spilled from a ruptured tanker spread in a circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast the area of the spill increasing when the radius of the spill is 60 feet. 7

(c) Evaluate the following:- 7

Let $x \to a \frac{xCos\,x-\ln(1+x)}{x^2}$

Q.2 (a) State and prove the Mean Value Theorem, give its geometrical interpretation. 1+7+2

(b) Prove that $\frac{x}{1+x} \le \ln(1+x) \le x$ 10

for -1 < x < 0, and for x > 0.

Q.3 (a) Evaluate $\int_{\phi}^{\lambda/L} \ln(\sin)dx$ 7

(b) Find the volume generated by revolving the area bounded by the parabola y2 = 8x and its rectum x = 2 about the y-axis. 6

(c) The cost of fuel in running a locomotive is proportional to the square of the speed and is $ 25 per hour for a speed of 40 km/hour. Other costs amount to $100 per hour, regardless of the speed. Find the speed which will make the cost per kilometer a minimum. 7

Q.4 (a) Approximate $\int_{\phi}^{1} x^{1/3}dx$, using trapezoidal and Simpson rules.

Compare the approximations to the actual values. 8

(b) Find the length of the cardiod $r=a(1-Cos\theta)$ and hence find its centroid. 12

Q.5 (a) Solve the following differential equations:-

(i) $\frac{dy}{dx}=\frac{x+y-1}{x-y+1}$ 6

(ii) $(2xy+y^2+3)dx+(x^2+2xy)dy=0$ 6

(b) Radium decomposes at a rate proportional to the quantity of radium present. It is found that in 25 years approximately 1.1% of a certain quantity of radium was decomposed. Determine approximately how long it will take for one half the original amount of radium to decompose. 8

SECTION "B"

Q.6 (a) Prove that monotonic decreasing sequence which is bounded below converges to its greatest lower bound. 7

(b) Let $\sum un$ to the series of positive and real numbers and 7

Let $n \to \infty \frac{un+1}{un}=l.$ Prove that series $\sum un$ converges if l < 1 and diverges if l > 1.

(c) Test the converges of the following series:- 6

$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\frac{7}{4.5.6}+\ldots\ldots\ldots$

Q.7 (a) Define an analytic function. If f is an analytic function of complex values, then show that $\frac{\partial f}{\partial Z}=0$. 6

(b) Evaluate the following integrals in the complex plane.

(i) $\int \frac{dZ}{1+Z^2}$ where C is that part of parabola y = 4x3 from A(2.0) to B(-2.0). 7

(ii) $\int \frac{9Z^2-iZ+4}{Z(Z^2+1)}dZ$, where C is a Circle $|Z|=2$. 7

Q.8 (a) Identify and sketch the curve: 6

153x2 - 192xy + 97y2 – 30x – 40y – 200 = 0

(b) If r (t) = a Cos t$^{\hat{i}}$ + a Sin t ĵ + $^{ct\hat{k}}$ is a circular helix, then show that radius of curvature of the helix is given by $\frac{a^2+c^2}{a}$, where a > 0 6

(c) If r(t) is a smooth space curve, then prove that its Torsion $\zeta(t)$ can be expressed as $\zeta(t)=\frac{[\underline{r}'(t)\times \underline{r}''(t)]\underline{r}''(t)}{\|\underline{r}'(t)\times \underline{r}''(t)\|^2}$ 8