

MATHEMATICS (OPTIONAL) PAPER-II

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS: 100

NOTE: Attempt five questions. Choose two questions from each section at least.

SECTION - A

1. Both the order and index of a sub-group of a finite group divide the order of the group. 20
2. If ϕ is a homomorphism of a group G into a group \bar{G} then,
 - (i) $\phi(e) = \bar{e}$, the unit element of \bar{G}
 - (ii) $\phi(x^{-1}) = \phi(x)^{-1}$ for all $x \in G$. 20
3. If F is a field of real numbers, Prove that $(1, 1, 0, 0)$, $(0, 1, -1, 0)$ and $(0, 0, 0, 3)$ in $F^{(4)}$ are linearly independent over F .
4. Give examples of:
 - (i) Field 10
 - (ii) Finite dimensional vector space 10

SECTION B

5. Give definitions and examples of:
 - (i) Metric space 10
 - (ii) Topological space 10
6. Define and give example of inner product space. 20
7. Find the matrix of the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ defined by:
 $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, x_1)$
with respect to the standard bases for \mathbb{R}^3 and \mathbb{R}^4 .
8. Define properties of determinants of order 2. 20