

**TIME ALLOWED: THREE HOURS**

**MAXIMUM MARKS: 100**

**NOTE: Attempt any FIVE questions.**

**Q.1 (a)** Two brands of cigarettes are compared to determine the variance of the difference  $D$  in the nicotine content of brands  $A$  and  $B$ . Let  $X$  and  $Y$  denote respectively the nicotine content of brand  $A$  with a s.d. of 3 mg and the nicotine content of brand  $B$  with a s.d. of 2.5mg. It is assumed that  $X$  and  $Y$  are independent. Find the mean, S.D. and coefficient of variation of  $D=X-Y$ . 20

**(b)** The following sums have been obtained from the frequency distribution of the variate  $X$  after making the substitution  $X = 10 + 5u$ .  $\Sigma f = 25$ ,  $\Sigma fu = -45$ ,  $\Sigma fu^2 = 306$ ,  $\Sigma fu^3 = -242$ ,  $\Sigma fu^4 = 1962$ . Calculate the mean, s.d.,  $m_3$  and  $m_4$  of the variate  $X$ .

**Q.2 (a)** State Chebyshev's inequality. The mean and s.d. of moderately skewed distribution are  $\bar{X} = 50$  and  $S = 10$  respectively. Compute the intervals  $\bar{X} \pm 2S$  and  $\bar{X} \pm 3S$ . Interpret them in the light of : 20

- (i) normal distribution and
- (ii) Chebyshev's inequality.

**(b)** In a certain community 10% of all adults over fifty have diabetes. A doctor correctly diagnoses 95% of all persons with diabetes as having the disease and incorrectly diagnoses 2% of all persons with diabetes as having the disease. What is the probability that a person diagnosed as having diabetes actually has the disease?

**Q.3 (a)** Two balls are selected at random from box containing 3 black, 2 red and 3 green balls. If  $X$  and  $Y$  are the number of black and red balls selected respectively, then find: 20

- (i) The joint distribution of  $X$  and  $Y$
- (ii) The marginal distributions of  $X$  and  $Y$
- (iii) Are  $X$  and  $Y$  independent?

**(b)** The joint density function of  $X$  and  $Y$  is

$$f(x,y) = (2x+y)/210, 2 < x < 6, 0 < y < 5.$$

- find (i) the marginal density function of  $X$  and  $Y$
- (ii) the conditional density function of  $Y$  given  $X$
- (iii) Are  $X$  and  $Y$  independent?

**Q.4 (a)** Under what conditions the binomial distribution  $f(x;n,p)$  is approximated by the Poisson distribution  $f(x;\lambda)$ ? Explain 20

Ten percent of the tools produced in a certain manufacturing process turns out to be defective. Find the probability that out of a Sample of 10, two will be defective by using.

- (i) the binomial distribution and
- (ii) the Poisson approximation to the binomial distribution.

**(b)** An oil drilling company selects various locations for drilling and their success or failure is independent from one location to another. Suppose that the probability of a success at any specific location is 0.25. What is the probability that a driller find the third success on sixth drilling.

**Q.5 (a)** The mean inside diameter of sample of 200 washers produced by a machine is 5.02 mm and the s.d. is 0.05 mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 4.96 to 5.08 mm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed. 20

**(b)** The m.g.f. of the  $N(\mu, \delta^2)$  with respect to the mean  $\mu$  is  $M_{x-\mu}(t) = e^{t^2 \delta^2 / 2}$ . Using this express for m.g.f., find  $\mu_{2r+1}$  and  $\mu_{2r}$ . Deduce the results for  $r=0, 1, 2$ .

**Q.6 (a)** Find the most plausible values of  $x$  and  $y$  from the equations. 20

$$2x+y = 0; 3x-2y = 0; -x+y = -2$$

**(b)** The number of bacteria ( $Y$ ) per unit volume in a culture after  $X$  hours is given in the following table. Fit a least square curve of the type  $y = ab^x$ . Estimate the value of  $Y$  when  $X=7$ .

$X$	0	1	2	3	4	5	6
$Y$	32	47	65	92	132	190	275

**Q.7 (a)** Define coefficient of determination ( $r^2$ ). 20  
What does it explain?  
Show that  $-1 \leq r \leq 1$ .

**(b)** In a trivariate distribution it is found that

$$\bar{X}_1 = 6.8, \bar{X}_2 = 7.0, \bar{X}_3 = 7.4,$$

$$S_1 = 1.0, S_2 = 0.80, S_3 = 0.60,$$

$$r_{12} = 0.60, r_{13} = 0.70, r_{23} = 0.65.$$

Find the partial regression coefficient  $b_{12.3}$  and  $b_{13.2}$ , the partial correlation coefficient  $r_{12.3}$ , the multiple correlation coefficient  $R_{1.23}$ , and the regression line of  $X_1$ , on  $X_2$  and  $X_3$ .

**Q.8 (a)** The population linear regression model is given by the equation  $Y_i = \alpha + \beta X_i + t_i$ . Explain the symbols  $\alpha$ ,  $\beta$  and  $t_i$  of the model and give the assumptions of the model. 20

**(b)** Write down the density function of the bivariate normal distribution  $f(x,y)$  with parameters  $\mu_1, \mu_2, \delta_1^2, \delta_2^2$ , and  $e$  (Correlation coefficient between  $X$  and  $Y$ ). How would you obtain the marginal density functions of  $X$  and  $Y$  without carrying out derivation? Write down these density functions of  $X$  and  $Y$ .