PUNJAB PUBLIC SERVICE COMMISSION
COMBINED COMPETITIVE EXAMINATION
FOR RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE, ETC.

MATHEMATICS (OPTIONAL) PAPER-I

TIME ALLOWED: 3 HOURS — MAXIMUM MARKS: 100

Note: Attempt any THREE questions from Section "A" and TWO questions from Sections "B".

Simple calculator is allowed.

SECTION "A"

Q.1: (a) State and prove the mean value theorem. Give its geometrical interpretation. (10)

(b) Draw a graph of the function "y" defined by the statement "y is the smallest positive number that makes x + y and integer." For what values of x is the function discontinuous? (10)

Q.2: (a) Evaluate the following: (10)

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin x-(\sin x)^{\sin x}}{1-\sin x+\ln \sin x}$$

(b) Rectangles are inscribed in a circle of radius r. Find the dimensions of the rectangle which has maximum area. (10)

Q.3: (a) Evaluate (10)

$$\int x(x^x)^x(2\log x+1)dx$$

(b) Find the length of the asteroid: $x = a\cos^3 t, : y = a\sin^3 t$ (10)

Q.4: (a) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by double integration. (08)

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sin x\, dx$ using 9 point Simpson's rule and 9 point Trapezoidal rule also compare them for accuracy. (12)

Q.5: Solve the following differential equation

i. $(2x\cos y+3x^2y)\,dx+(x^3-x^2\sin y-y)\,dy=0,\quad y(0)=2$ (7)

ii. $e^x\left(1+\frac{dy}{dx}\right)=xe^{-y}$ (7)

(b) Assume that oil spilled from a ruptured tanker spread in circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast the area of the spill is increasing when the radius of the spill is 60 feet? (6)

SECTION "B"

Q.6: (a) Prove that every bounded sequence has at least one limit point. (7)

(b) Show that the sequence $< S_n >$ defined by (6)

$$S_1=1, S_{n+1}=\frac{4+3s_n}{3+2s_n}, n\in N$$

is convergent and find its limit.

(c) Test the convergence the following series (7)

$$\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+\ldots\ldots\ldots\ldots\ldots\ldots$$

Q.7: Evaluate integrals

i. $\left|\oint \bar{z}\right|$ dz where C is a semi unit circle. (6)

ii. $\oint \frac{6z^2-2z+5}{(z-1)^3}dz$. Where C encloses the point z = 1. (7)

iii. $\oint \frac{e^{2z}+\sin z}{(z+1)^4}dz$. Where C encloses the point z = -1

Q.8: (a) If r (t) is a smooth space curve, them prove that Torsion ζ (t) can be expressed as $\zeta(t)=\frac{[r'(t)\times r''(t)]r'''(t)}{\|r'(t)\times r''(t)\|^2}$ (10)

(b) The section of a cone whose vertex is P and guiding curve is the ellipse $x^2/a^2+y^2/b^2=1$, z = o by the plane x = 0 is a rectangular hyperbola. (8)

Show that the Locus of the vertex P is ellipse $x^2/a^2+\frac{y^2+z^2}{b^2}=1$ (12)