PUNJAB PUBLIC SERVICE COMMISSION
COMBINED COMPETITIVE EXAMINATION FOR RECRUITMENT TO THE POSTS OF PROVINCIAL MANAGEMENT SERVICE, ETC.

MATHEMATICS (OPTIONAL) PAPER-II

TIME ALLOWED: 3 HOURS **MAXIMUM MARKS: 100**

Note: Attempt 'Five' questions is all by selecting at least 'two' questions from each section. All questions carry equal marks. Calculator is allowed.

SECTION-A

Q.1: (a) Define 'Cyclic Group'. Show that every 'subgroup' of a 'Cyclic group' is 'Cyclic'. (10)

(b) Let '*H*' and '*K*' be two finite subgroups of a group 'G' whose orders are relatively prime. Prove that $H \cap K = \{e\}$ (10)

Q.2: State and prove 'Fundamental theorem of Homomorphism'. (20)

Q.3: (a) Determine '*k*' so that the vectors $(1, -1, k - 1)$, $(2, k, -4)$, $(0, 2 + k, -8)$ in $\mathbb{R}^3$ are linearly dependent. (10)

(b) Suppose, *U*, and, *W*, are distinct four dimensional subspaces of a vector space, *V*, of dimension six. Find the possible dimension of $U \cap W$. (10)

Q.4: (a) Define 'Rings' and 'Fields' with examples. (08)

(b) Does the set of all symmetric "3 x 3" matrices form a vector space or not? If your answer is "Yes" determine the dimension and find a basis. Justify your answer as well. (12)

Q.5: (a) Let X =] 0, 1 [for $x, y \in X$ define $d: X \times X \rightarrow \mathbb{R}$ by (10)

$$d(x,y) = \left|\frac{1}{x} - \frac{1}{y}\right|$$

Show that, *d*, is a metric on, *X*.

(b) Let, $(X, \Im_1)$, and, $(X, \Im_2)$, be two topological spaces, then $\Im_1 \cap \Im_2$, is also a topology on, X. (10)

Q.6: Using "Gram-Schmidt process of ortho-normalization", transform the basis {(1,1,1), (0,1,1), (0,0,1)} into an ortho-normal basis. (20)

Q.7: Apply the Gauss-Seidel iteration to the following system. Do "08" steps (iterations). Take the initial guess be $x^{(0)} = y^{(0)} = z^{(0)} = 1$ (20)

$$4x - y = 21$$

$$-x + 4y - z = -45$$

$$-y + 4z = 33$$

Q.8: Given the matrix (20)

$$A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

Find the matrix '*P*' such that, $P^T AP$, is a diagonal. Find also the diagonal matrix.