

**PUNJAB PUBLIC SERVICE COMMISSION**  
**COMBINED COMPETITIVE EXAMINATION**  
**FOR RECRUITMENT TO THE POSTS OF**  
**PROVINCIAL MANAGEMENT SERVICE, ETC.**

**STATISTICS (OPTIONAL) PAPER-I**

**TIME ALLOWED: 3 HOURS**

**MAXIMUM MARKS: 100**

**Note:** Attempt Any FIVE questions.

Calculator is allowed.

**Q.1:** (a) The first 3 moments of a distribution about the value 2 of the variable are 1, 16, and - 40. Find the mean, variance and  $m_3$ . What can be said about the skewness of the distribution? Also find the first 3 moments about the point zero.

(b) State the properties of arithmetic mean. (20)

**Q.2:** (a) A random variable  $X$  has mean 8 and variance 9 and an unknown probability distribution. Find

(i)  $P[-4 < X < 20]$

(ii)  $P[|X-8| \geq 6]$

(b) An instructor gives her class a set of 10 problems with the information that the final examination will consist of a random selection of 5 of them. If a student has figured out how to do 7 of the problems, what is the probability that he will answer correctly (i) all 5 problems (ii) at least 4 of the problems? (20)

**Q.3:** (a) Given the joint density

$$f(x,y) = x + y, \quad 0 < X < 1, \quad 0 < y < 1$$

$$= 0, \text{ elsewhere}$$

Find (i) the marginal densities of  $X$  and  $Y$  (20)

(ii) the conditional density function of  $Y$  given  $X$

(iii)  $Cov(X,Y)$

(b) Let  $X$  be a continuous random variable with probability density function given as:

$$f(x) = 6 \times (1-x), \quad 0 \leq x \leq 1$$

$$= 0, \text{ elsewhere}$$

Find the mean, mode, harmonic mean of this distribution.

**Q.4:** (a) The probability that a person will die from a certain respiratory infection is 0.002. Find the probability that fewer than 5 of the next 2000 so infected will die.

(b) Find the probability that a person flipping a coin gets:

(i) the third head on the seventh flip

(ii) the first head on the fourth flip.

(c) The probabilities are 0.4, 0.2, 0.3 and 0.1 respectively, that a delegate to a certain convention arrived by air, bus, car or train. What is the probability that among 9 delegates selected at this convention, 3 arrived by air, 3 arrived by bus, 1 arrived by car and 2 arrived by train? (20)

**Q.5:** (a) A man goes by car to his office, and the route through the city centre takes him on the average 27 minutes with a standard deviation of 5 minutes. With the opening of a new ring road, the man can bypass the congestion of the city centre, but the journey now takes on the average 29 minutes with a standard deviation of 2 minutes. Assuming both journey times are normally distributed, determine which road is better if the man has (i) 28 minutes, and (ii) 32 minutes to reach his office for an appointment.

(b) Suppose that the death rate from malaria is 20%, find the probability that the number of deaths in a particular village is between 70 and 80 (inclusive) out of 500. (20)

**Q.6:** (a) If  $X$  and  $Y$  have a bivariate normal distribution with parameters  $\mu_x, \mu_y, \sigma_x^2, \sigma_y^2$  and  $P_{xy}$ , write the marginal density functions of  $X$  and  $Y$  and the conditional density function of  $Y/X=x$ .

(b) Fit a curve of the form  $Y=aX^b$  to the following data on the unit cost of producing certain electronic components and the number of units produced.

Lot Size ( $X$ )	50	100	250	500	1000
Unit Cost ( $Y$ )	108	53	24	9	5

Use the result to estimate the unit cost for a lot of 400 components.

(20)

**Q.7:** (a) State the principle of least squares and apply it to solve the following equations:

$$X+7Y = 17, \quad 2X-Y=0, \quad 3X-2Y= -1$$

(b) State the properties of the least squares regression line.

**Q.8:** (a) Given the following information find the regression equation of  $X_3$  on  $X_1$  and  $X_2$ . Also find  $R_{3.12}$  and  $r_{13.2}$ .

$$\bar{X}_1 = 20, \quad \bar{X}_2 = 36, \quad \bar{X}_3 = 12, \quad S_1 = 1.0, \quad S_2 = 2.0, \quad S_3 = 1.5,$$

$$r_{12} = -0.20, \quad r_{13} = 0.40, \quad r_{23} = 0.50 \quad (20)$$

(b) Prove that the correlation coefficient is independent of the origin and scale.